# The static model of labour supply

## Labour Economics ECON 87100

Miles Corak

Department of Economics
The Graduate Center, City University of New York

@milescorak
milescorak.com

Lecture 2

# The consumer's problem

## and the full income budget constraint

maximize some utility function $U(X, L)$

subject to two constraints: $PX = WH + Y$, and $T = L + H$

- the Full Income Budget Constraint

$$PX + WL = WT + Y = F$$

- where we think of leisure as another commodity to be consumed, having a real price $\frac{W}{P}$, so that $X = -\frac{W}{P}L + \frac{F}{P}$

# The consumer's problem

## and the underlying assumptions of the static model

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## and the underlying assumptions of the static model

Within the context of a one-period model

1. linear budget constraint
   - taxes, overtime rates
   - fixed costs of working
2. continuous budget constraint
3. single person household
4. intensive margin

# The consumer's problem

## and the underlying assumptions of the static model

All intertemporal linkages are abstracted from

1. no borrowing or saving so that current consumption equals current (after tax) income
2. the wage rate is fixed, ignoring human capital accumulation through work experience and its impact on future wages
3. no evolution in preferences
4. fertility, marriage, and other demographics are taken as given in a single person household

# 1. The Lagrangian and the first order conditions

$$\max U(X, L) \text{ subject to } F = PX + WL$$

$$\max U(X, L) + \lambda(F - PX - WL) \text{ implies the FOC:}$$

$$U_X - \lambda P = 0$$

$$U_L - \lambda W = 0$$

$$F - PX - WL = 0$$

$$\text{implying } \frac{U_L}{U_X} = \frac{W}{P}, \text{ or } MRS_{XL} = w$$

# 2. Solving the first order conditions

The FOC may be solved for commodity and leisure demand functions, the latter defines a labour supply function

$$X = X(P, W, Y)$$

$$L = L(P, W, Y)$$

$$H = T - L(P, W, Y)$$

# 3. Comparative statics

- we are asking what happens to the optimal values of X, L, and H when the exogenous variables change
- since $H = T - L(P, W, Y)$, $dH/d\theta = -dL/d\theta$ where $\theta$ represents any one of these exogenous variables
- we can imagine examining:
  1. How a caeterius paribus change in P effects the equilibrium values of X,L, and H
  2. How a caeterius paribus change in Y effects the equilibrium values of X,L, and H
  3. How a caeterius paribus change in W effects the equilibrium values of X,L, and H
- we can get these predictions by finding partial derivatives of the FOC.

# 3. Comparative statics

## changes in W and the impact on labour supply

$$\delta L/\delta W = S_{LL} + H\delta L/\delta Y$$

where $S_{LL} < 0$ and $H\delta L/\delta Y > 0$ $if \delta L/\delta Y > 0$ , that is if L is a normal good.

The sign of $\delta L/\delta W$ is ambiguous.

$$\delta H/\delta W = -\delta L/\delta W$$

$$\delta H/\delta W = -S_{LL} - H(\delta L/\delta Y)$$

$$\delta H/\delta W = S_{HH} + H(\delta H/\delta Y)$$

where $\delta H/\delta Y = -\delta L/\delta Y$ and $S_{HH} > 0$. $H(\delta H/\delta Y) < 0$ if $\delta H/\delta Y < 0$

Theory does not predict the sign of the slope of the labour supply function.

# The law of demand revisited

## when the consumer is also a seller

The standard theory of consumer demand assumes the consumer's income is fixed in terms of money

- exceptions to the law of demand are rare and inconsequential
- a possibility only if the good is inferior, makes up a large proportion of the budget, and the substitution effect is weaker than the income effect

# The law of demand revisited

## when the consumer is also a seller

But what if the consumer comes to the market not just as a buyer, but also as a seller?

- when the price of the good changes, the substitution effect is the same, reducing demand (and increasing supply) when the price increases
- but the income effect is not the same, a price increase makes a seller better off, leading to increased demand (and lower supply) when the good is normal

For buyers, the income and substitution effects work in the same direction; for sellers, in the opposite direction.

- when it comes to leisure we may reasonably expect the "law of demand" not to hold. The income effects could be large since sellers obtain a large fraction of income from the commodity they sell.
- *The impact of wage rate changes on labour supply is an empirical issue.*

# Why are we interested in the issue?

## Labour supply elasticities are central to the design of tax/transfer programs

The more elastic labour supply in response to changes in after-tax wage rates, the lower the optimal tax rate.
For example, the optimal income tax rate for the top income bracket of a progressive system is given as: $\tau = \frac{1}{1+a \times e}$

- where $e$ is the labor supply elasticity, $a$ is the Pareto parameter indicating the (inverse of) income dispersion in the top income bracket $a = \frac{z_m}{z_m - z}$, ($z$ is the top income cut-off, and $z_m$is the average income for those above it). A flat tax—with no brackets and a single tax rate—would be characterized by $z = 0$ and $a = 1$, making the optimal tax rate $\tau = \frac{1}{1+e}$ .
- in other words, the top rate increases as inequality at the top rises
- generally $a$ has been estimated to be around 1.5 to 2 for typical top brackets in countries like the US.

# Why are we interested in the issue?

## Labour supply elasticities are central to the design of tax/transfer programs

TABLE 1
OPTIMAL TOP BRACKET TAX RATES FOR DIFFERENT LABOR SUPPLY ELASTICITIES

| Labor supply elasticity ($e$) | Optimal top-bracket tax rate ($\tau$) | | |
|---|---|---|---|
| | $a = 1.50$ | $a = 1.67$ | $a = 2.0$ |
| 2.0 | 25% | 23% | 20% |
| 1.0 | 40% | 37% | 33% |
| 0.67 | 50% | 47% | 43% |
| 0.5 | 57% | 54% | 50% |
| 0.3 | 69% | 67% | 63% |
| 0.2 | 77% | 75% | 71% |
| 0.1 | 87% | 86% | 83% |
| 0.0 | 100% | 100% | 100% |

*Note*: These rates assume the government places essentially no value on giving extra income to the top earners.

Source: Keane (2011). "Labor Supply and Taxes: A Survey," *Journal of Economic Literature*, Table 1, page 964.

# The estimation strategy

## often labour supply functions are directly specified and estimated with least squares

$$lnH_{i,t} = \alpha + \beta lnw_{i,t}(1-\tau) + \beta_y y_{i,t} + \varepsilon_{i,t}$$

- we have cross-sectional data at time $t$ on individuals $i$=1...N
- lower case signifies real variables ($w$=$W/P$ and $y$=$Y/P$)
- $\tau$ is the appropriate tax rate
- $\varepsilon_{i,t}$ is a stochastic term capturing individual heterogeneity, reflecting supply side (taste based) shocks and measurement errors

# The estimation strategy

## often labour supply functions are directly specified and estimated with least squares

$$lnH_{i,t} = \alpha + \beta lnw_{i,t}(1-\tau) + \beta_y y_{i,t} + \varepsilon_{i,t}$$

- $\beta$ and $\beta_y$ are the parameters of interest
  - $\beta$ is the Marshallian elasticity of hours with respect to the (net) wage rate
  - it can be used with $\beta_y$ and the Slutsky equation in elasticity form, $\frac{w}{H}\frac{\delta H}{\delta w} = \frac{w}{H}S_{HH} + \frac{wH}{y}\left[\frac{y}{H}\frac{\delta H}{\delta Y}\right]$, to back out the Hicksian elasticity as $S_{HH} = \beta - w_{i,t}(1-\tau)\beta_y$

# Hey, what could possibly go wrong?

# Hey, what could possibly go wrong?

## 1. Endogeneity of wages and non labour income

The crucial Right Hand Side variables may be endogenous because of omitted variables.

- tastes for work may be correlated with wage rates and with non labour income
- those choosing long hours may be more productive and command a higher wage rate, setting up a positive correlation between wages rates and $\varepsilon_{i,t}$
- they could also save more, build up more assets so that non labour income is also correlated with the stochastic term

# Hey, what could possibly go wrong?

## 1. Endogeneity of wages and non labour income

The crucial Right Hand Side variables may also be endogenous because of simultaneity bias.

- it is not clear if we are estimating a demand curve, a supply curve, or some sort of reduced form
- there is a need to understand the source of the variation in wage rates and non labour incomes across individuals in the sample
- the estimation strategy needs plausible exclusion restrictions, for example taste-based factors that change wage rates but are not in the set of the individual characteristics determining an individual's productivity, and hence wage rate

# Hey, what could possibly go wrong?

## 2. Taxes may introduce bias, and kinks in budget sets

Tax systems are generally progressive, with transfers to low income individuals that taper off to a break-even income, and then with marginal rates that increase with income.

- this implies that an individual's tax rate (as well as non labour income) depend upon wage rates and hours of work.
- the estimating function becomes

$$lnH_{i,t} = \alpha + \beta lnw_{i,t}(1 - \tau_i(w_{i,t}, H_{i,t})) + \beta_y y_{i,t}(w_{i,t}, H_{i,t}) + \varepsilon_{i,t}$$

- implying that that wages and non labour income depend on hours
- a taste for more work places the individual in a high income tax bracket leading to a negative correlation with the stochastic term
- tax systems also introduce kinks in the budget constraint, making the slope indeterminate

# Hey, what could possibly go wrong?

## 3. Measurement error in wage rates and non labour income

The crucial Right Hand Side variables may be measured with error

- wage rates may be measured with error, and correlated with the stochastic term leading to attenuation bias
- wage rates are commonly constructed by dividing annual earnings by annual hours of work, what Keane (2011) calls the "denominator" problem
  - the left hand side variable is negatively correlated with the calculated wage rate for spurious reasons—if the reported hours are unusually high, the derived wage rate will be unusually low—lending a downward bias to the estimated coefficient

# Hey, what could possibly go wrong?

## 4. Wage rates are unobserved for non participants

Selection bias may arise if information on those outside of the labour force is not taken into account

- non participation implies that the reservation wage is greater than the prevailing market wage
- labour market participants may have lower reservation wages, so that those for whom wage rates are observed have a higher realization of $\varepsilon_{i,t}$, a higher taste for work at a given wage rate
- Heckman (1974) is a simultaneous equations strategy to the identification issue, taking account of all the information from the data, whether or not an individual is a labour force participant. But his formulation requires a priori exclusion restrictions, and makes assumptions about functional form of the likelihood function.

# Hey, what could possibly go wrong?

## 5. Dynamics and life cycle considerations cast their shadow

Non labour income is not simply wealth, but also asset income derived from savings and life cycle patterns in consumption

- assets evolve over the life cycle in an inverted U-shape, and are not necessarily a good indicator of lifetime wealth

Wage rates are determined in part by work experience

- wage rates are assumed fixed, reflecting human capital decisions of the past (presumably formal schooling and past work experience), but decisions to work could reflect the impact it will have on future wages

# Estimated elasticities for men

## vary according to Keane (2011, page 1042) from 0.02 to 1.32, and average 0.31

TABLE 6
SUMMARY OF ELASTICITY ESTIMATES FOR MALES

| Authors of study | Year | Marshall | Hicks | Frisch |
|---|---|---|---|---|
| *Static models* | | | | |
| Kosters | 1969 | −0.09 | 0.05 | |
| Ashenfelter-Heckman | 1973 | −0.16 | 0.11 | |
| Boskin | 1973 | −0.07 | 0.10 | |
| Hall | 1973 | n/a | 0.45 | |
| Eight British studies[a] | 1976–83 | −0.16 | 0.13 | |
| Eight NIT studies[a] | 1977–84 | 0.03 | 0.13 | |
| Burtless-Hausman | 1978 | 0.00 | 0.07–0.13 | |
| Wales-Woodland | 1979 | 0.14 | 0.84 | |
| Hausman | 1981 | 0.00 | 0.74 | |
| Blomquist | 1983 | 0.08 | 0.11 | |
| Blomquist-Hansson-Busewitz | 1990 | 0.12 | 0.13 | |
| MaCurdy-Green-Paarsch | 1990 | 0.00 | 0.07 | |
| Triest | 1990 | 0.05 | 0.05 | |
| Van Soest-Woittiez-Kapteyn | 1990 | 0.19 | 0.28 | |
| Ecklof-Sacklen | 2000 | 0.05 | 0.27 | |
| Blomquist-Ecklof-Newey | 2001 | 0.08 | 0.09 | |
| *Dynamic models* | | | | |
| MaCurdy | 1981 | 0.08[b] | | 0.15 |
| MaCurdy | 1983 | 0.70 | 1.22 | 6.25 |
| Browning-Deaton-Irish | 1985 | | | 0.09 |
| Blundell-Walker | 1986 | −0.07 | 0.02 | 0.03 |
| Altonji[c] | 1986 | −0.24 | 0.11 | 0.17 |
| Altonji[d] | 1986 | | | 0.31 |
| Altug-Miller | 1990 | | | 0.14 |
| Angrist | 1991 | | | 0.63 |
| Ziliak-Kniesner | 1999 | 0.12 | 0.13 | 0.16 |
| Pistaferri | 2003 | 0.51[b] | | 0.70 |
| Imai-Keane | 2004 | 0.40[e] | 1.32[e] | 0.30–2.75[f] |
| Ziliak-Kniesner | 2005 | −0.47 | 0.33 | 0.54 |
| Aaronson-French | 2009 | | | 0.16–0.61 |
| Average | | 0.06 | 0.31 | 0.85 |

# Estimated elasticities for women

## are larger than for men according to Keane (2011, page 1070)

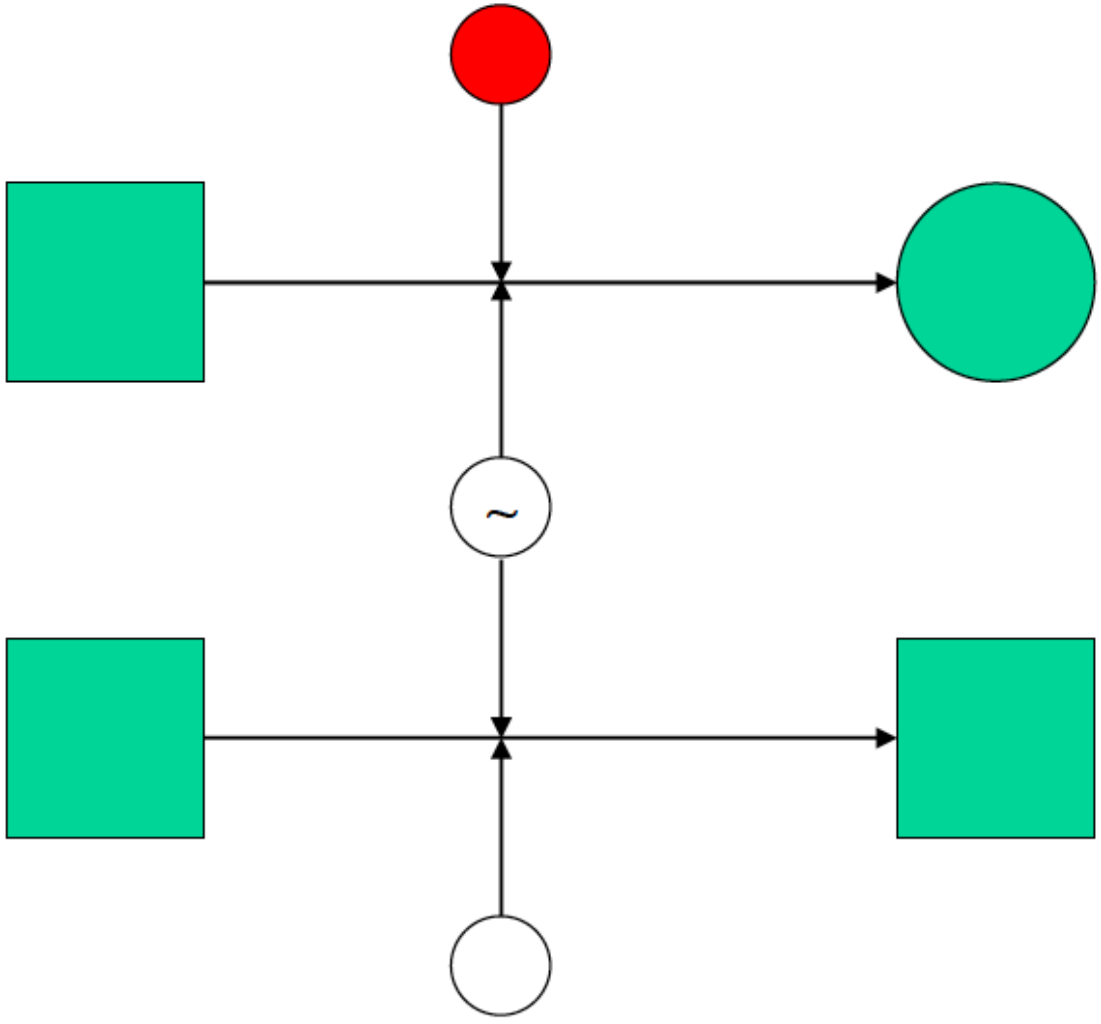
TABLE 7
SUMMARY OF ELASTICITY ESTIMATES FOR WOMEN

| Authors of study | Year | Marshall | Hicks | Frisch | Uncompensated (dynamic) | Tax response |
|---|---|---|---|---|---|---|
| *Static, life-cycle and life-cycle consistent models* | | | | | | |
| Cogan | 1981 | 0.89[a] | | | | |
| Heckman-MaCurdy | 1982 | | | 2.35 | | |
| Blundell-Walker | 1986 | −0.20 | 0.01 | 0.03 | | |
| Blundell-Duncan-Meghir | 1998 | 0.17 | 0.20 | | | |
| Kimmel-Kniesner | 1998 | | | 3.05[b] | | |
| Moffitt | 1984 | | | | 1.25 | |
| *Dynamic structural models* | | | | | | |
| Eckstein-Wolpin | 1989 | | | | 5.0 | |
| Van der Klauuw | 1996 | | | | 3.6 | |
| Francesconi | 2002 | | | | 5.6 | |
| Keane-Wolpin | 2010 | | | | 2.8 | |
| *Difference-in-difference methods* | | | | | | |
| Eissa | 1995, 1996a | | | | | 0.77–1.60[b] |

*Notes:*
[a] = Elasticity conditional on positive work hours.
[b] = Sum of elasticities on extensive and intensive margins.

# Discerning causality

## a randomized trial as the "ideal" solution



# Discerning causality

## 1. advantages of randomized controlled trials

1. Address the biases that arise from the selection problem
   - selection arises from the fact that there is missing information on the factors that affect both the participation in the program and the outcome
   - by randomly denying treatment a RCT creates variation in exposure to the treatment that is independent of the decision to participate
   - observational studies (of the kind Keane 2011 reviews) rely on collecting more information, or using more sophisticated statistical methods to model the selection process

# Discerning causality

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# Discerning causality

## 1. advantages of randomized controlled trials

1. Address the biases that arise from the selection problem
2. Experiments can be designed to measure treatments not previously observed
3. The causal results are easy to communicate

# Discerning causality

## 2. disadvantages (or challenges) of randomized controlled trials

1. What are the disadvantages of RCTs?
2. What are the challenges in implementing a RCT in the social sciences?

# Large Scale Experiments

## 1. 1970s vintage RCTs

1. Negative Income Tax experiments were designed to examine the labour supply consequences of a Guaranteed Annual Income
   - characterized by a lump sum grant, a tax-back rate, and a break-even income level.
   - our budget constraint changes to $PX = G + (1-\tau)(WH + Y)$, which involves a positive subsidy to the individual of $G - \tau(WH + Y)$ when $\frac{G}{\tau} > (WH + Y)$
   - $\frac{G}{\tau}$ is the break-even income, so that a higher $G$ or a lower $\tau$ raise the break-even income and increase the scope of the program
   - the change in work hours induced by the program is $dH = -\tau W S_{HH} + \frac{\delta H}{\delta Y}[G - t(WH + Y)]$, given that $dW = -\tau W$ and $dY = G - \tau Y$

# Large Scale Experiments

## 1. 1970s vintage RCTs

1. Negative Income Tax experiments were designed to examine the labour supply consequences of a Guaranteed Annual Income in the US
2. The Seattle and Denver Income Maintenance Experiments had a variety of lump sum grants ranging from \$3,800 to \$5,600, tax back rates from 0.5 to 0.8, and break-even incomes from \$5,429 to \$12,000 across a number of groups of households (each numbering around 75), and apparently randomized between controls and treatments.

# Large Scale Experiments

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3. A similar program in Canada (Dauphin Manitoba), that has recently been revisited with intergenerational data on a host of other outcomes.

# Large Scale Experiments

## 2. 1990s vintage RCTs

### The Self-Sufficiency Project in Canada during the 1990s

- Card and Hyslop (2005) describe and analyze a much more generous, and in many dimensions better managed RCT, with Michalopoulos *et al.* (2002) offering much more detail.
- A more targeted program addressed to long-term Income Assistance receipts who were single parents, involving over 2,800 individuals in each of the treatment and control groups
- The treatment was conditional on full-time employment, and offered a generous wage subsidy of one-half the difference between earnings and a benchmark
- Think of it as a negative income tax with $\tau = 0.5$, and a grant above prevailing welfare benefits, but conditional on working at least 30 hours/week.
- An important requirement is that members of the treatment group had up to one-year to find full-time work and establish their entitlement. If not their eligibility for the subsidy was lost.

TABLE I

KEY FEATURES OF THE SSP RECIPIENT DEMONSTRATION

**A. Program eligibility**

- Eligibility limited to single parents who have received Income Assistance (IA) for at least 12 months.
- Sample members drawn from IA registers in British Columbia and New Brunswick, with random assignment between November 1992 and February 1995.
- 2,858 single parents assigned to the program group; 2,826 assigned to the control group.

**B. Program features**

- Subsidy payments available to program group members who work at least 30 hours per week (over a 4-week or monthly accounting period) and earn at least the minimum wage.
- Subsidy recipients become ineligible for IA.
- Subsidy equals one-half of the difference between actual earnings and an earnings benchmark, set at $2,500 per month ($30,000 per year) in New Brunswick and $3,083 per month ($37,000 per year) in British Columbia in 1993, and adjusted for inflation in subsequent years.
- Subsidy payments are unaffected by unearned income or the earnings of a spouse/partner, and are treated as regular income for income tax purposes.
- Subsidy payments are available for 36 months from time of first payment. Payments are only available to program group members who successfully initiate their first supplement payment within 1 year of random assignment.
- Once eligible, program group members can return to IA at any time. Subsidy is re-established when an eligible person begins working full time again.
- Employers are not informed of SSP status. Program group members apply for subsidy payments by mailing copies of payroll forms.

Source: Card and Hyslop (2005, Table 1, page 1726).

# Large Scale Experiments

## 2. 1990s vintage RCTs

The Self-Sufficiency Project in Canada during the 1990s was very well managed

- Control and treatment groups had similar characteristics, and randomization was implemented
- Reporting involved a combination of administrative data on Income Assistance and earnings, and also a series of surveys that followed up participants for five years
- It overcame many of the criticisms and limitations of the Seattle and Denver experiments, and was much more generous

# Large Scale Experiments

## 2. 1990s vintage RCTs

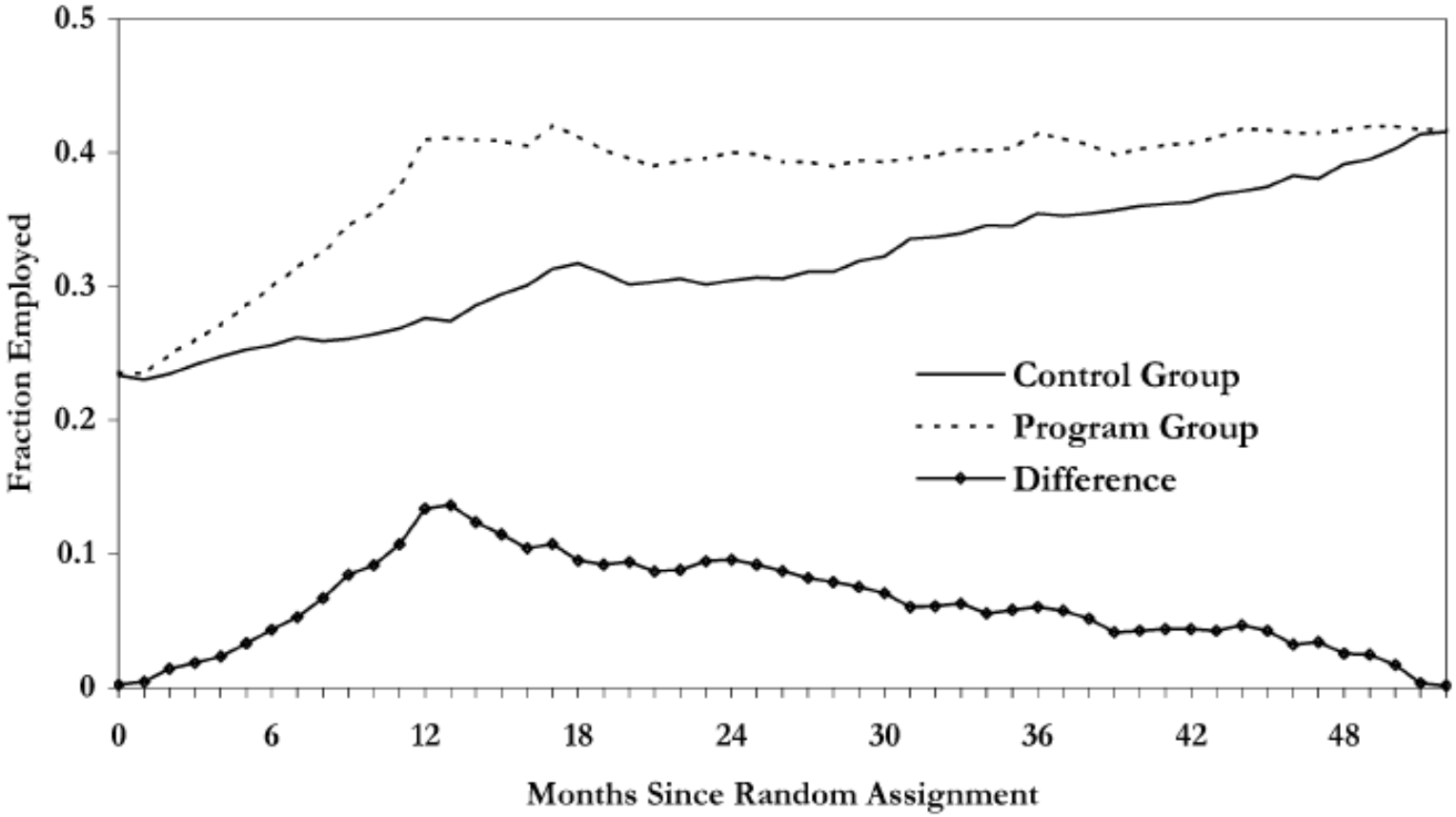


FIGURE 3.—Monthly employment rates.

Source: Card and Hyslop (2005, Figure 1, page 1734).

# Advantages and disadvantages of large scale RCTs

1. How do the Negative Income Tax experiments, or more recently The Self-Sufficiency project, reflect both the advantages and challenges of Randomized Controlled Trials?
2. What do they really tell us about the responsiveness of labour supply to changes in the net wage?
3. The labour supply literature is full of things we can do, but shouldn't. And large scale RCTs are, in some sense, things we should do, but can't. So what are the take-aways from all this, and what is the research agenda that follows?
4. Where is Dauphin Manitoba?


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